



K24U 0058

Reg. No. : .....

Name : .....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – OBE – Regular/  
Supplementary/Improvement) Examination, April 2024

(2019 to 2021 Admissions)

CORE COURSE IN MATHEMATICS

6B10 MAT : Real Analysis – II

Time : 3 Hours

Max. Marks : 48



PART – A

Answer **any four** questions. **Each** question carries **one** mark.

(4×1=4)

1. Give an example of a step function defined on  $[1, 4]$ .
2. Write norm of the partition  $P = (0, 5, 7, 9, 10)$  of  $[0, 10]$ .
3. State additivity theorem.
4. Define Gamma function.
5. Define  $\varepsilon$ - neighborhood of a point  $x_0$  in a metric space  $(S, d)$ .

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

(8×2=16)

6. State non-uniform continuity criteria for a function  $f : A \rightarrow \mathbb{R}$ .
7. Using an example, show that product of monotonic increasing functions need not be increasing.
8. Let  $f(x) = x^2$ ,  $x \in [0, 5]$ . Calculate Riemann sum with respect to the partition  $P = (0, 1, 3, 5)$ , take tags at the left end point of the subintervals.
9. Show that value of the integral of a Riemann integrable function is unique.

P.T.O.



10. If  $f$  is a Riemann integrable function and  $k \in \mathbb{R}$ , show that  $kf$  is Riemann integrable

$$\text{and } \int_a^b kf = k \int_a^b f.$$

11. Evaluate  $\int_1^{\infty} \frac{1}{x} dx$ .

12. Show that  $B(m, n) = B(n, m)$ .

13. Compute  $\Gamma(-1/2)$ .

14. Find pointwise limit of the sequence of functions  $(x^n)$  for  $x \in [0, 1]$ .

15. Define a metric  $d$  on a set  $S$ .

16. State Cauchy criterion for convergence for sequence of functions.

### PART – C

Answer **any four** questions. **Each** question carries **four** marks.

(4×4=16)

17. Define uniformly continuous function. Show that  $f(x) = x^2$  is not uniformly continuous on  $[0, \infty)$ .

18. Show that Riemann integrable functions defined on  $[a, b]$  are bounded on  $[a, b]$ .

19. Show that if  $f, g \in R[a, b]$ , then  $f + g \in R[a, b]$  and  $\int_a^b (f + g) = \int_a^b f + \int_a^b g$ .

20. Evaluate  $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ .

21. From the definition of beta function, derive  $B(m, n) = \int_0^1 \frac{y^{n-1}}{(1+y)^{m+n}} dy$ .

22. Derive  $\Gamma(n) = \int_0^{\infty} [\log(1/t)]^{n-1} dt$ .

23. Show that a sequence of bounded functions  $(f_n)$  defined on a set  $A$  converges uniformly on  $A$  to a function  $f$  if and only if  $\|f_n - f\| \rightarrow 0$ .



PART – D

Answer **any two** questions. **Each** question carries **six** marks.

(6×2=12)

24. a) Define a Lipschitz function. Show that Lipschitz functions are uniformly continuous.

b) Show that not every uniformly continuous function is a Lipschitz function.

25. State and prove Fundamental theorem of calculus (1<sup>st</sup> form).

26. Show that  $B(m,n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$ .

27. a) Using an example, show that pointwise limit of a sequence of continuous functions need not be continuous.

b) Given that  $(f_n)$  is a sequence of continuous functions defined on a set A such that  $(f_n)$  converges uniformly to a function f defined on A. Prove that f is continuous on A.

